From Debt to Equity in Technology Investment: Did Policy Makers Get it Right?

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Abstract

We explore the consequences of a series of policies enacted at the end of the 1970s which encouraged technology investment to move from a debt to an equity contract. We capture the technological focus of such investment through an endogenous growth model, this time using a model similar to the one in Rivera-Batiz and Romer (1991). We capture debt contracts through a net-worth multiplier, as in Bernanke and Gertler (1989), and equity contracts through an issuance cost, as in Covas and Haan (2011). We find that the change in contracts can account for a large increase in investment over the period and a break in the correlation between TFP growth and the returns to production capital. It can also partially explain the relative variance in TFP growth between decades, and the model corresponds to certain elements of the 1973 recession.

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1 Introduction

Venture capital deals typically take the form of a private investment in a company in return for equity in the firm. However, during the period from 1959 to 1979, the United States federal government invested directly into small firm financing in the technology sector. In 1958, the Small Business Administration was given the authority to charter "Small Business Investment Companies." These SBICs financed firms through debt due to their bureaucratic structure. By 1963, these firms provided over 75% of investments into new technologies. The shift from SBICs to private venture capital firms is credited largely to the change in the ERISA's "prudent man" rule in 1979. This change explicitly allowed pension funds to invest in venture capital. Coinciding with a decrease in capital gains tax the previous year, funding in new technology companies shifted from debt to equity relatively quickly and can be attributed to an exogenous policy change.[1]

A number of authors have argued that the form of contract is a significant determinant of venture capital funding. Issues such as investment duration, non-financial contributions, moral hazard, and risk-return profiles have gained much attention. Specifically, both costly state verification and monitoring of current investments have been identified as significant contractual issues with venture capital, each having a special role in debt and equity contracts, respectively.[2]

This paper investigates whether a change from a debt to an equity contract in funding new technology ventures around the year of 1980 had any effect on the general equilibrium behavior of the market. The role of venture capital in funding technological advancement is central to our analysis, in that it allows for growth rates of the economy to vary endogenously and based on incentivized behavior. The general intuition behind this analysis is that the structural role of technology investment through its impact on total factor productivity causes a magnification and propagation of the effects of a change in contract type.

[1]The historical information above comes from Gompers (1994)

[2]Kaplan and Stramberg (2001) is one particularly strong example, and Da Rin et al. (2011) offers a complete survey of the use of contracts in venture capital on a micro level.

We build a model that incorporates R&D costs, similar to Rivera-Batiz and Romer (1991), into a dynamic stochastic general equilibrium framework. We then incorporate two types of financial contract in the technology sector, in turn. As we will see, both types of financial contacts have some sort of contractual cost to them, forcing a break in the Modigliani-Miller theorem.[3] Using the financial accelerator of Bernanke and Gertler (1989), we incorporate a debt contract into the technology sector using a formulation similar to Carlstrom and Fuerst (1997).[4] This debt contract is in response to moral hazard under asymmetric information, in which the returns to an investment are determined prior to the realization of private information to a borrower, and the cost of the contract comes through the state-dependent monitoring costs, where monitoring only happens when a borrower claims default.[5] For an equity contract, we appeal to Covas and Haan (2011) in incorporating quadratic equity costs to mimic the expense of selecting and monitoring investments.

We find that the models incorporating the different contracts each fit the data better for their respective periods than a model in which contractual limitations are not present. Furthermore, incorporating the contractual limitations for each period helps explain a number of stylized facts:

1. the decreased variance in TFP from 1960-1969 relative to the periods immediately before and after the policy was implemented;[6]

2. the depth and duration of the recession of 1973-74;

3. the increased funding to technological assets in the years 1984–2008; and

4. a reversal in the correlation between TFP growth and interest rates occurring around the year 1980.

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[3]Modigliani and Miller (1958) show that the capital structure of a firm does not affect the returns to capital investment in the firm. The theorem fails to hold when financial contracts have costs associated with them, such as maintenance or information costs.

[4]The key differences being that CF used a debt contract in an economy in which technology was given as exogenous and that the limitations from debt contract applied universally across all assets. We focus the debt contract on the technological sector, where the returns to the asset act as the incentive for investment in new technology.

[5]Townsend (1979) shows that a debt contract is the optimal solution to the costly state verification problem.

[6]The exclusion of the 1970s from this stylized fact is explained below.

Decade	Variance
50s	12.43
60s	8.93
70s	14.83
80s	11.76

Table 1: Variance of quarterly utilization-adjusted TFP over various decades. Source: Fernald (2014)

First, in the decade immediately following the authorization of the Small Business Administration to issue debt contracts, the variance of utilization-adjusted total factor productivity (TFP) growth reduced significantly. As shown in Table 1, quarterly calculations of utilization-adjusted TFP growth are considerably lower in the 1960s than in the 1980s or 1950s. This is in line with the mechanics of our model, which allow a much larger deviation in resources allocated to technology development in an equity contract than when a debt contract is used, following a technology shock of similar magnitude. This is because the risk is borne entirely by the entrepreneur in the debt contract, with the lender only responding through changes in the terms of lending. As the entrepreneur is less risk-averse and more impatient than the lender, much of the brunt of the shock will be absorbed by the entrepreneur, rather than passed on through the terms of the loan.

While the model corresponds with relative changes in measured variance of utilization-adjusted TFP between the 1950s, 1960s, and 1980s, the 1970s do not follow the predictions of our model. While the period which we characterize through a debt contract for technological investment includes the 1970s, this decade exhibits a high variance in the quarterly utilization-adjusted TFP growth level, which corresponds to an equity contract in our model. However, the 1970s was a decade characterized by a high-level of supply-side volatility, due to shocks in the oil supply and the ultimate end of the Bretton Woods international financial system. Therefore, we believe that the comparison of the 1970s was not like-for-like in the underlying shocks, making any comparison with surrounding decades tenuous. In addition, the high and volatile inflation of

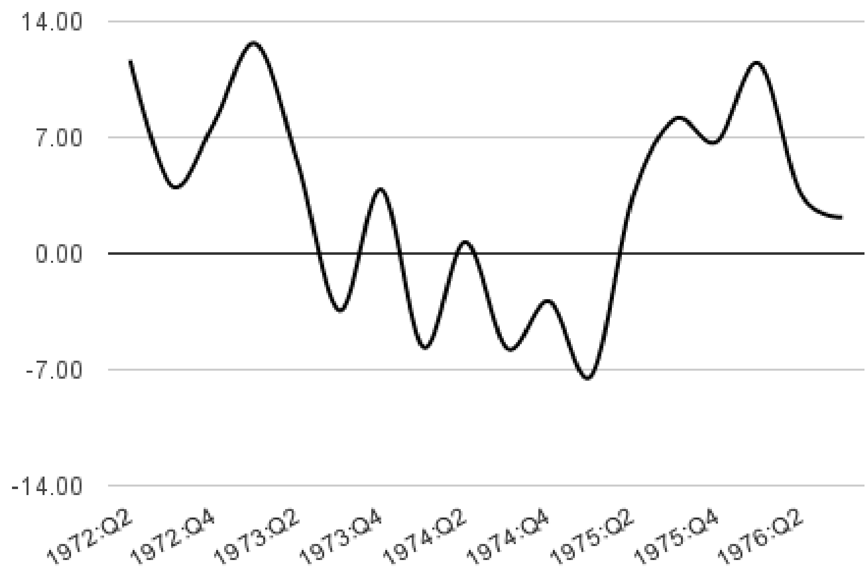


Figure 1: Real GDP Growth in the 1972 Recession

the 1970s begs an explanation that includes a monetary mechanism, which is not included in our model. Thus, while our model predicts the relative variances of technology growth in the 50s, 60s, and 80s, we believe the 1970s to be an outlier that can be explained through other means.

Despite the limitation of our model to explain the oil-related volatility of the 1970s, our model does bear some relevance to a particular incident in the decade. The recession of the early 70s in the United States was one marked by persistently low GDP growth, as shown in Figure 1. While the overall level of GDP growth never dipped below -5% in any given quarter, the period from the pre-recession peak in the first quarter of 1973 to a level above the long-term average in 1975 is particularly long and was marked by 5 quarters of negative real growth rates. Given a strong negative shock during a period in which technology investment occurs under a debt contract, our model predicts that the recession would be protracted and long in duration, similar to other models exhibiting the net-worth accelerator. In our model, the debt contract forces the returns to investment in technology to remain low in subsequent periods due to the borrowing constraints of the entrepreneurs in the tech sector, depressing any technologically-led recoveries. Indeed, the utilization-adjusted TFP growth referenced above shows below-average levels for 1972-74.

More importantly, our model predicts a large increase in the amount of private investment in technological development when an equity contract is used. This is due to the shared risk between entrepreneur and investor in a debt contract, which allows the investor to allocate his assets to the technology projects at exactly the time in which she wishes to increase investment.

Finally, the dynamics of our model match the observed break in correlation between returns to investment in production equities and total factor productivity growth following the change in policy encouraging technological investment to follow an equity contract. We see this first by considering the utilization-adjusted total factor productivity growth figures in Fernald (2014), and comparing them to the capital input levels from the same dataset. As theoretical competitive returns to capital in our model are $(1-\alpha)\frac{Y}{K_Y}$, we can compare these utilization-adjusted TFP growth rates with the theoretical returns to capital. These values are negatively correlated from 1950 to 1977, and positively correlated from 1980 to 2000, at values of -0.10 and 0.19, respectively. Moreover, comparing these same utilization-adjusted TFP values with the returns to equities found in Shiller (1992) and updated in Shiller (2014), this shift in correlations is even stronger between the same two periods, at -0.30 and 0.42, respectively.

There is a relatively large body of research loosely related to the analysis at hand. The relationship between type of finance and economic development is well-established, with more developed "frontier" economies relying more on equity markets than developing economies, which rely more on debt. Levine and Zervos (1998), Bencivenga and Smith (1991), and Bencivenga et al. (1995) discuss the relationship between the type of finance and the level of development in various economies, with the latter specifically focusing on transaction costs. In addition, Agénor and Aizenman (1997) and Boyd and Smith (1998) discuss a costly state verification model in the context of economic development. Another strand of related literature focuses on the broader economic implications of venture capital. Many of these focus on more narrow employment and economic activity of venture capital funded companies, while a few focus on the broader economic implications of venture capital. Two pertinent examples of this type of analysis are Kortum and Lerner (2000) and Keuschnigg (2004).

The only other paper known to this author dealing explicitly with the general equilibrium implications of contracting in the context of technology growth is Reiss and Weinert (2005). This paper is similar to ours in that it incorporates moral hazard into a general equilibrium endogenous growth model. However, the focus is not on venture capital; the type of growth described is different from ours, and the overall focus is on the effects on inequality of endogenous growth. Fernandez-Villaverde et al. (2003) deal with the implications of financial intermediation and entrepreneurship in the macroeconomic context, but they stop short of using endogenous technological change as the main mechanism for growth in their model.

2 The model

In this section, we build a baseline model from which we compare the effects of using a debt contract versus an equity contract in creating new technology. The debt contract is adapted from Carlstrom and Fuerst and exhibits a financial multiplier. The intuition is that the net-worth of an entrepreneur can fetch a proportional amount of funding from the market. However, the entrepreneur also absorbs the idiosyncratic risks and returns from the economy, causing his net-worth to fluctuate. As the level of overall funding is tied to the net-worth of the entrepreneur at any period, the absorbed losses and gains from aggregate shocks cause a magnification to the fluctuations in the overall funding for the entrepreneur's project, and shocks persist as they are propagated by fluctuations in the entrepreneur's net-worth. As the economy suffers, the market price of capital will also increase, hurting the terms of the contract for the entrepreneur, and further limiting the amount of capital available for the project.

The equity contract has some similar elements to the debt contract.[7] The entrepreneur's net-worth is still central to the terms of the contract—the entrepreneur determines how much equity he wishes to raise, given that he must provide a just compensation for any outside funding. However, a negative shock to the economy is now shared between both parties, as failing projects lead to commensurate losses to both the entrepreneur and lender.

The assumption that a debt contract necessitates the risk burden being shouldered by borrowers is central to the concept of debt, in which the repayment amount is determined at issuance. In contrast, the shared ownership of equity is fundamentally one of risk sharing as the costs and benefits of ownership are distributed across equity holders.

In addition, the switch from debt to equity also exhibits a move from a state-dependent monitoring cost to a quantity-dependent issuance or governance cost. This, coupled with the more equitable risk burden between parties described above, drives our results.

The cost structure of each asset class is less fundamental to the definition of the class, but the differences are still intuitively defensible. First, the role of costly state verification is important in debt, as the asymmetry between lender and borrower creates a moral hazard for the borrowers to lie. This, coupled with the observed costs of monitoring in bankruptcy and the result that this type of contract being optimal in the case of moral hazard makes monitoring costs an attractive form for modeling the cost of the contract. Likewise, in the absence of information asymmetries, the constraints to equity come in the form of issuance and governance costs, with regulatory compliance, the organization of shareholder voting, and the issuance of dividends as the dominating costs of issuing equity. The first two of these costs, regulatory compliance and the organization of shareholder voting, would arguably increase with the size of the number of investors, which we represent through a quadratic cost of issuance of equity.

Furthermore, the overall level of investment is closely related to the marginal productivity of the project itself, meaning that while decreases in the entrepreneur's net-worth will result in a lower overall investment due to the costs of the contract being related to the level of equity

[7] Our equity contract is related to that of Covas and Haan (2011).

created, the terms of the contract may end up more favorable to the entrepreneur with a low net-worth (depending on the strength of the equity costs). As a result, the entrepreneur's returns are countercyclical (or less pro-cyclical than a debt contract), and the equity contract dampens the impact of a negative shock.

3 Production

To include the effects of venture capital, we build a model of increasing product variety, as in Rivera-Batiz and Romer (1991). This model incorporates incentivized growth through the development of new intermediate goods used in the production process. That is, our production process includes a variety of intermediate capital goods entering through a Dixit-Stiglitz aggregator:

$$Y_t = L_t^{1-\alpha} \int_0^{A_t} x_{i,t}^{\alpha} di \tag{1}$$

where Y_t is production, L_t is labor used in production, x_i is an intermediate capital good of type i, and A_t is the number of intermediate goods at time t. Tpaperhis can be used to derive an equilibrium interest rate, r_t, and profits, π_t:

$$r_t = \alpha^2 \frac{Y_t}{K_{Y,t}} \tag{2}$$

$$\pi_t = \alpha(1-\alpha)\frac{Y_t}{A_t} \tag{3}$$

We assume, for simplicity, symmetry over the intermediate goods, so the overall level of capital dedicated to production, $K_{Y,t}$ can be divided among the intermediate capital goods. That is, for all i, we represent $x_{i,t}$ through a generalized $\bar{x}_t$, which is equal to $\frac{K_{Y,t}}{A_t}$. Thus, (1) can be simplified as:

$$Y_t = L_t^{1-\alpha} A_t \left(\frac{K_{Y,t}}{A_t}\right)^{\alpha}$$
$$= (A_t L_t)^{1-\alpha} K_{Y,t}^{\alpha} \quad (4)$$

4 Technology and patents

At the heart of this model is a project for the development of a new intermediary good. For this, our economy contains a continuum of technological entrepreneurs, each of which has a proposed project for a new intermediate good to be used in production. Each project uses capital and the existing stock of technology to develop new intermediate goods. The projects differ in their effectiveness in creating new technology, using capital as the primary input.[8]

$$a_{i,t+1} = \zeta k_{A,i,t}^{1-\lambda} A_t^{\lambda} \theta_i \quad (5)$$

where θ_i represents the effectiveness of entrepreneur, i, and the subscript, a, on capital denotes that this is a portion of the capital stock allocated to the technology sector. λ represents the diminishing returns to capital and existing technology in the creation of new technology, and is bound by $0 < \lambda < 1$. Across this paper, lower case letters represent individual variables and upper case variables represent aggregate variables.[9]

Furthermore, we assume that the timing of this investment requires that the level of capital investment made to a specific technology be determined before any of the agents realize the level

[8] We use the Rivera-Batiz and Romer model rather than the model from Romer (1990), as the former uses capital as the primary input of production. The efficiency of the size of the project is crucial in comparing the two types of finance. We require the input to technology to exhibit diminishing returns to scale in order for the project to have an efficient size, and a stationary variable such as human capital or labor could not lead to balanced growth with diminishing returns to scale.

[9] The idea of an entrepreneur creating an individual technology is a technical but useful abuse of notation, as once the technology is created, it immediately and automatically contributes to the overall level of technology through the non-rival nature of ideas. However, the technology is individual in the sense that it is created by an individual entrepreneur and in that the entrepreneur can then create an exclusive right to that technology for the purposes of production.

of θ_i. This eliminates any concerns that private information can lead to signaling or adverse selection through the level of investment requested in the project.

In aggregate, the individual abilities of each entrepreneur aggregate to a constant θ_{agg}, which we normalize to one.[10] Our aggregate technology production function is therefore,

$$A_{t+1} - A_t = \zeta K_{A,t}^{1-\lambda} A_t^{\lambda}. \tag{6}$$

where new technology is now given as the difference between the future and current levels of A, and $K_{A,t}$ is the aggregate level of capital allocated to the development of new technology. Combined with $K_{Y,t}$, this gives our entire capital stock, K_t. At the individual level, the entrepreneur creates new technology using the overall stock of technology, A, so we can ignore the effects of previous technology created by any specific entrepreneur.

As capital is the only input for creating a new intermediate good, the cost of creating the good is the price of capital times the amount of capital dedicated to technology. We call this initial investment, i_t. Once the project is complete and a new intermediate good, $A_{t+1} - A_t$, has been created, $\zeta K_{A,t}^{1-\lambda} A_t^{\lambda} q_t$ assets are created in the form of patents, where q_t is the price of a *new* patent.

To pay for this capital, the entrepreneur provides his own net-worth, n_t, and borrows the remaining amount, $i_t - n_t$, from households in the market. The new assets are divided up between the lender and entrepreneur, depending on the type of contract, as described below.

The behavior of the entrepreneur drives the dynamics of the investment in the technology project for this model. Fluctuations in the entrepreneur's net-worth drive the market price and level of capital in a way that is dependent on the type of contract used. However, the contracts are similar in that the entrepreneur faces a mechanism design problem in which she optimizes the terms of the contract to maximize her own return, subject to a participation constraint of the

[10] Individual realizations of this θ are central to the debt contract, while we are able to completely overlook the existence of theta in the equity contract due to normalizing the aggregate level to one. While agents are heterogeneous over their individual level of theta, the expected value of theta is normalized to one, implying the effects of this heterogeneity are distributional rather than contributing to aggregate levels.

household providing the rest of the capital. As we will see, both contracts exhibit a participation constraint that requires returns to the household to be equal to the opportunity cost of investing in the productivity sector. Given this inelastic return to the household, the entrepreneur varies the proportion of returns she receives and the overall size of the project in order to maximize her return given the restraints of each contract. Thus, a lack of net-worth for the entrepreneur manifests itself in a higher proportion of proceeds going to the household, but also in a lower investment in the project. This lower investment determines the propagation mechanism, as the funding for the project determines the growth rate of the economy.

We discuss the debt contract and the equity contract in turn.

5 Debt

As with Bernanke et al. (1999), the existence of the entrepreneur is significant in that it provides a "net-worth multiplier," causing the aggregate investment to fluctuate with the net-worth of the entrepreneur, a binding restriction. The key is in the private information available to the entrepreneur, giving an edge in investing in this type of asset for this agent. As a result, the returns will be higher, and the entrepreneur will want to invest his entire net-worth into the asset in each period. However, the terms of the contract will fluctuate with the net-worth of the entrepreneur, which will drive cyclical fluctuations in the overall investment in this asset class.

At the heart of the debt contract is the idea that the effectiveness of the entrepreneur's technology project is private information known only to the entrepreneur. The lender can discover the value of the entrepreneur's effectiveness level by paying a monitoring cost proportional to the size of the project. However, this monitoring cost is a deadweight loss, and the household and entrepreneur desire to build a contract in which the lender refrains from paying that cost as often as possible.

At the beginning of each period, the entrepreneur works and receives, x_t, in wages which he contributes to his stock of patents for a net-worth of,

$$n_t = x + z_t p_t \tag{7}$$

which he invests into his own technology project along with capital raised from the household. The entrepreneur is endowed with a minimal amount of capital in each period to ensure that the net-worth at the beginning of a period is positive. In addition, our shocks are calibrated to ensure that aggregate net-worth never dips below zero.

At the end of the period, the technology project produces patents, according to (6), which are divided between the entrepreneur and the lender, with $\zeta k_{a,t}^{1-\lambda} A_t^{\lambda} q_t f(\bar{\theta})$ going to the entrepreneur, $\zeta k_{a,t}^{1-\lambda} A_t^{\lambda} q_t g(\bar{\theta}_t)$ to the lender, and the rest are lost in the process of monitoring the signal of the entrepreneur. We can express this through the identity, $f(\bar{\theta}) + g(\bar{\theta}) = 1 - \mu\Phi(\bar{\theta})$, where $\Phi(\bar{\theta})$ is the probability that the lender will pay the monitoring cost.

The entrepreneur's decision from period to period is to optimize his utility from consumption, similar to the household. However, since he has superior returns to the household from the technology enterprise, he is only concerned with one asset, patents.

Once the entrepreneur receives his own return from the project, he consumes part and holds the rest for the next period:

$$p_t z_{t+1} = q_t \zeta k_{a,t}^{1-\lambda} A_t^{\lambda} f(\bar{\theta}_t) - c_t^e \tag{8}$$

As it is useful to express transition functions in terms of state variables, we incorporate (7) to give us,

$$p_t z_{t+1} = (x + z_t p_t)^{1-\lambda} q_t \zeta \left(\frac{k_{a,t}}{n_t} \right)^{1-\lambda} A_t^{\lambda} f(\bar{\theta}_t) - c_t^e \tag{9}$$

Since the entrepreneur has superior information, he has the bargaining power in the formation of the contract. Specifically, we treat him as the optimizing agent in determining the terms of payment, choosing which level of repayment he promises to give in a successful project. There are two opposing effects on the entrepreneur increasing the threshold signal for a successful project. First, by increasing the promised return to the lender for a successful project, the entrepreneur is directly paying the lender a higher share of the returns. However, by increasing this level, the entrepreneur must also bear a larger share of the initial investment, lowering the potential value of the project. By promising a high return, the entrepreneur also increases the likelihood that any project will be unsuccessful, as it will be more difficult to meet the threshold for a project to return the promised investment.

Specifically, the entrepreneur optimizes his own return $\zeta k_{a,t}^{1-\lambda} A_t^{\lambda} q_t f(\bar{\theta})$, subject to a participation constraint of the lender. That is, the lender will only offer money if the expected return is greater than potential other uses. Thus, the maximization problem is:

$$\max_{k_{a,t}} q_t \zeta k_{a,t}^{1-\lambda} A_t^{\lambda} f(\bar{\theta})$$

$$q_t \zeta k_{a,t}^{1-\lambda} A_t^{\lambda} g(\bar{\theta}_t) \geq (k_{a,t} - n_t)(1 - \delta + r_t) \tag{10}$$

We limit ourselves to interior solutions in which this will always bind, allowing us to state the participation constraint as a necessary portion of the initial investment:

$$\frac{n_t}{k_{a,t}} = 1 - q_t \zeta k_{a,t}^{-\lambda} A_t^{\lambda} g(\bar{\theta}_t)(1 - \delta + r_t)^{-1} \tag{11}$$

The first order conditions from the optimization are:

$$(1-\lambda) q_t \zeta k_{a,t}^{-\lambda} A_t^{\lambda} f(\bar{\theta}) = -\frac{f'(\bar{\theta}_t)}{g'(\bar{\theta}_t)}((1 - \delta + r_t) - (1-\lambda) q_t \zeta k_{a,t}^{-\lambda} A_t^{\lambda} g(\bar{\theta}_t)) \tag{12}$$

where the symbol, $x'(*)$, represents a derivative. The left side of the equation above can be described as the benefit to the entrepreneur of increasing $k_{a,t}$ at a given level of $\bar{\theta}_t$. The right side represents the cost of increasing the capital level. The term inside the brackets represents the marginal cost of increasing the size of the project net of the benefit from the increase in the project. The fraction represents the change in the proportion going to each agent given a change in the size of the project.

Defining $\Omega(\bar{\theta}_t)$ as the marginal increase in the monitoring cost associated with an increase in the project size,[11] it is useful to re-arrange (12) to obtain the form,

$$q(1-\lambda)\zeta k_{a,t}^{-\lambda} A_t^{\lambda} - (1 + r_t - \delta) = \Omega(\theta)\zeta k_{a,t}^{-\lambda} A_t^{\lambda} q_t \tag{13}$$

Here, we see the optimal condition for the entrepreneur drives a wedge between the marginal return to capital from the technology project and the marginal return to capital from the production sector. The left side is the difference between the two marginal productivities. As omega tends to zero on the right side, this difference disappears. Thus, the cost of monitoring effectively chokes off technological development in the model and lowers the growth rate.

Since the contract only lasts a single period, the entrepreneur does not consider the constraints of the contract explicitly in his optimization. Thus, combining the entrepreneur's transition function, (9), with the participation constraint, (11), we get a transition function entirely in terms of the entrepreneur's state variable, z, the choice variable, c^e, and variables considered exogenous to the entrepreneur.

$$p_t z_{t+1} = (x + z_t p_t) \frac{q_t \zeta k_{a,t}^{1-\lambda} A_t^{\lambda} f(\bar{\theta})}{1 - \delta + r_t - q_t \zeta k_{a,t}^{1-\lambda} A_t^{\lambda} g(\bar{\theta}_t)} - c_t^e \tag{14}$$

The entrepreneur's preferences are given differently from the consumer's. Instead of CRRA preferences, the entrepreneur is considered risk neutral, allowing his lifetime consumption to fluctuate more than a risk-averse consumer would. Also, we stipulate that the entrepreneur is more impatient in his consumption decisions, with a time preference given by $\tilde{\beta} \equiv \beta\gamma$. Some

[11]Specifically, it is defined as $\mu\left(\Phi(\bar{\theta}_t) + \Phi'(\bar{\theta}_t)\frac{f(\bar{\theta}_t)}{f'(\theta_t)}\right)$

authors interpret this increased impatience as a higher probability of death or likelihood of switching from being an entrepreneur to a household.[12]

The entrepreneur's intertemporal decision is given by the optimization:

$$\max_{c_t^e} \mathbf{E}_t \sum_{t=0}^{\infty} \beta^t c_t^e$$
$$\text{s.t. } p_t z_{t+1} = (x + z_t p_t)^{1-\lambda} q_t \zeta \left(\frac{k_{a,t}}{n_t}\right)^{1-\lambda} A_t^{\lambda} f(\bar{\theta}_t) - c_t^e \tag{15}$$

As usual, his decisions are represented through an Euler equation:

$$p_t = \beta\gamma \mathrm{E}_t \left[p_{t+1} q_{t+1} \zeta \left(\frac{k_{a,t+1}}{n_{t+1}}\right)^{1-\lambda} A_{t+1} f(\bar{\theta}_{t+1}) \right] \tag{16}$$

We now show the explicit form of $f(\bar{\theta}_t)$ and $g(\bar{\theta}_t)$, given by the standard debt contract as initially proposed by Townsend (1979).

5.1 The debt contract

The lender offers to lend money to the project in return for receiving his money back with interest, r^k, at the end of the project. Any additional proceeds from the project go to the entrepreneur. However, some projects will not be profitable enough to pay back the interest. There will be a threshold below which the the project will not yield enough money to pay the lender the promised amount. We index this threshold by the signal corresponding to the marginal project, $\bar{\theta}_t$. For all projects with a signal below this critical level, the lender recoups the entire value of the project, less a monitoring cost, $\mu\Phi(\bar{\theta}_t)$, to verify the amount.[13]

$$q_t \zeta k_{A,t}^{1-\lambda} A_t^{\lambda} \bar{\theta}_t = (1 + r_t^k)(i_t - n_t) \tag{17}$$

[12] For example, Bernanke et al. (1999)

[13] Note that *all* projects still receive the same amount of funding, as the signal is not known to the lender until the time of default.

We follow CF in assuming that the household is risk-neutral in its consideration of intra-temporal risk across projects. In contrast, the household is risk-averse in its inter-temporal allocation across sectors. Again, we appeal to the same intuition of CF that there could be risk-neutral financial intermediaries that manage the debt contract with the entrepreneurs. We abstract away from these intermediaries for simplicity rather due to any intuitive distinction.

The entrepreneur therefore receives,

$$
\begin{aligned}
q_t \zeta k_{A,t}^{1-\lambda} A_t^{\lambda} f(\bar{\theta}_t) &= q_t \zeta k_{A,t}^{1-\lambda} A_t^{\lambda} \int_{\bar{\theta}_t}^{\infty} \theta \phi(\theta) d\theta - (1 - \Phi(\bar{\theta}_t))(1 + r_t^k)(i_t - n_t) \\
&= q_t \zeta k_{A,t}^{1-\lambda} A_t^{\lambda} \left[\int_{\bar{\theta}_t}^{\infty} \theta \phi(\theta) d\theta - (1 - \Phi(\bar{\theta}_t))\bar{\theta}_t \right] \qquad (18)
\end{aligned}
$$

while the lender receives,

$$
\begin{aligned}
q_t \zeta k_{A,t}^{1-\lambda} A_t^{\lambda} g(\bar{\theta}_t) &= q_t \zeta k_{A,t}^{1-\lambda} A_t^{\lambda} \int_{0}^{\bar{\theta}_t} \theta \phi(\theta) d\theta + (1 - \Phi(\bar{\theta}_t))(1 + r_t^k)(i_t - n_t) - \mu \Phi(\bar{\theta}_t) \\
&= q_t \zeta k_{A,t}^{1-\lambda} A_t^{\lambda} \left[\int_{0}^{\bar{\theta}_t} \theta \phi(\theta) d\theta + (1 - \Phi(\bar{\theta}_t))\bar{\theta}_t - \mu \Phi(\bar{\theta}_t) \right] \qquad (19)
\end{aligned}
$$

We define $\int_0^{\infty} \theta \phi(\theta) d\theta \equiv 1$ to make the relationships above represent the proportion of capital independent of the level of capital generated.

Adding the entrepreneur's share to the lender's, we obtain the useful result,

$$
f(\bar{\theta}_t) + g(\bar{\theta}_t) = 1 - \mu \Phi(\bar{\theta}_t) \qquad (20)
$$

While $f(\bar{\theta}_t)$ and $g(\bar{\theta}_t)$ represent the proportions of the newly created patents going to the entrepreneur and lender, respectively, the shares themselves do not add to one. Instead, we have a monitoring cost $\mu \Phi(\bar{\theta}_t)$, which is lost in each period due to the need for the lender to verify the state in the case of bankruptcy.

Using (20), we can eliminate $g(\bar{\theta}_t)$ from (12):

$$1 - \delta + r_t = p_t q_t \zeta k_{a,t}^{1-\lambda} \left(1 - \mu\Phi(\bar{\theta}_t) + \mu\phi(\bar{\theta}_t)\frac{f(\bar{\theta}_t)}{f'(\bar{\theta}_t)}\right) \tag{21}$$
$$\equiv p_t q_t \zeta k_{a,t}^{1-\lambda} h(\bar{\theta}_t)$$

In addition to allowing a reduced form of the first order conditions, (21) offers an intuitively appealing description of the behavior of q_t. The entrepreneur's optimal changes to the promised return to the lender are directly offset by the price of the new patent to allow the expected return from the project compete with the market in acquiring capital. While the entrepreneur isn't directly changing the price of new patents, his constraint on the supply of patents by altering the terms of the contract effectively decides this price. Conditions (21) and (10) completely describe the contract, $(q_t,\, \bar{\theta}_t)$, which depends entirely on the overall supply of foregone consumption to the project, and the proportion of that supply originating from the household.

6 Equity

We compare the contract to one in which the entrepreneur promises an investor a share of the overall return from the project, regardless of the outcome of the project. The structure of the equity contract is similar to that of the debt contract in that the entrepreneur chooses the size of the contract and the proportion of proceeds accruing to the household subject to the same participation constraint for the household. However, instead of a state-dependent contract, the entrepreneur now creates a more simple contract in which the share accruing to each agent is independent of the private signal of the entrepreneur. Similarly, the monitoring costs associated with the contract also are no longer state-dependent. Instead, there is a quadratic cost of issuing equity, similar to Covas & Den Haan (2012).

As mentioned above, this quadratic cost can be interpreted as the cost of governance of an enterprise given the amount of equity issued. Practically, this is quadratic to represent the increasing marginal cost of governance in issuing more shares. This can be due to disclosure requirements, the increased risk of demands from active shareholders, or the increased costs of holding shareholder meetings. In effect, the inclusion of having a linear marginal cost of equity is also to allow the models to be comparable, as the debt contract has a linear increased cost of monitoring for each unit of probability that the contract will be in default. This probability, in turn, is a function of the higher returns a borrower must pay to attract capital for a larger project. Thus, the tradeoff between both contracts is between a larger project and the marginal cost of attracting more capital. We have tried to keep this marginal cost as simple as possible.

Recall that the efficiency of a debt contract came through the existence of a private signal known by the entrepreneur. As such, the entrepreneur could expect a return from the project above market rates, and the efficiency of the debt contract arose in the ability for the household to receive market returns while monitoring only part of the time. By not conditioning the return to the household on the success of projects, the efficiency of this system is potentially gone. This poses two problems—it eliminates the market power of the entrepreneur to dictate the terms of the contract, and it potentially allows for the investor to increase his investment infinitely without any decrease in efficiency. Both these problems are eliminated by adding a cost of issuing equity to outside investors that is quadratic in the amount of equity issued.

As above, the entrepreneur brings a net-worth to the technology project, consisting of his assets from previous projects and his labor wage from the beginning of the period:

$$n_t = x + z_t p_t \tag{22}$$

where the labor is set-up the same way as in the debt contract.

Again, the entrepreneur enlists the help of the household to provide capital to the project, only now as an investment. At the end of the period, the technology project produces patents, which are divided between the entrepreneur and the lender. However, now the division of patents at the end of the period does not depend on the returns from individual projects. Instead, the household investor receives a fixed share, s_t, of the project, as agreed at the time of investment.

The entrepreneur's problem is now simply:

$$\max_{s_t}(1-s_t)\left[q_t\zeta k_{a,t}^{1-\lambda}A_t^{\lambda}\right]-\frac{as_t^2}{2}$$
$$\text{s.t. } s_t\left[q_t\zeta k_{a,t}^{1-\lambda}A_t^{\lambda}\right]=(1+r_t-\delta)(k_{a,t}-n_t) \tag{23}$$

where $k_{a,t}$ and n_t remain the amount invested in a technology project and entrepreneur's net-worth, respectively. As with the debt contract, the contract is described by a pair of parameters (q_t, s_t), given by the participation constraint, (23), and the first order condition to the maximization, given by,

$$k_{a,t}-n_t=\frac{q_t(1-\lambda)\zeta k_{a,t}^{-\lambda}A_t^{\lambda}-(1+r_t-\delta)}{a} \tag{24}$$

This term is straightforward: the marginal cost of increasing the amount of equity issued, $a(k_{a,t}-n_t)$, must equal the marginal benefit of that equity, which is the numerator on the right side. As the share of returns accruing to the household investor only enters the optimization linearly, the point at which the marginal return from increasing the shares equals the marginal cost will be independent of the proportion of the project going to each person.[14]

Similar to (13), (24) represents a wedge driving apart the marginal returns from the different uses of capital. As a tends to zero, any difference between the marginal benefit between using capital for technology and production would result in the entrepreneur raising an infinite amount of capital for his project. As a increases, capital is restricted from the technology project, driving up the marginal return from the project. In this sense, the equity contract is similar to capital adjustment costs used in some RBC and New Keynesian models. However, in this model, the amount of equity required for a given level of investment depends on the level of net-worth of the investor. As the net-worth increases, these "equity issuance costs" decrease, allowing the gap

[14] As mentioned before, the optimal condition would also be independent of the size of the overall project, $k_{a,t}$, if it also were not for the decreasing returns to scale in the input to technology project.

between projects to close. Thus, in order to see the dynamic effects of this wedge, it is necessary to assess the progression of the entrepreneur's net-worth.

Progressing as we did with the debt contract, the entrepreneur uses the return from the project for his own consumption,

$$p_t z_{t+1} = (1-s) q \zeta k_{a,t}^{1-\lambda} A_t^{\lambda} - a s^2 - c_t^e \tag{25}$$

As it is useful to express transition functions in terms of state variables, we incorporate (7) to give us,

$$p_t z_{t+1} = (x + z_t p_t)^{1-\lambda} (1-s_t) q_t \zeta \left(\frac{k_{a,t}}{n_t} \right)^{1-\lambda} A_t^{\lambda} - a s_t^2 - c_t^e \tag{26}$$

From (23), we get the identity,

$$\frac{k_{a,t}}{n_t} = \frac{1 + r_t - \delta}{1 + r_t - \delta - s_t q_t \zeta k_{a,t}^{-\lambda} A_t^{\lambda}} \tag{27}$$

We can now set up our entrepreneur's inter-temporal problem as,

$$\begin{aligned} &\max_{c_t^e} \mathbf{E}_t \sum_{t=0}^{\infty} \beta^t c_t^e \\ \text{s.t. } & p_t z_{t+1} = (x + z_t p_t)^{1-\lambda} (1-s_t) q_t \zeta \left[\frac{k_{a,t}}{n_t} \right]^{1-\lambda} A_t^{\lambda} - a s_t^2 - c_t^e \end{aligned} \tag{28}$$

An interior solution is described by,

$$p_t = \beta \gamma (1-\lambda) \mathbf{E}_t t_{t+1}^{-\lambda} p_{t+1}^{1-\lambda} q_{t+1} (1 - s_{t+1}) \zeta \left[\frac{k_{a,t+1}}{n_{t+1}} \right]^{1-\lambda} A_{t+1}^{-\lambda} \tag{29}$$

A negative productivity shock to the economy would cause the rental price of capital, r_t, to increase. In order to raise the same amount of equity, the entrepreneur would have to increase the household's share of the returns from the project. The entrepreneur responds by issuing slightly less equity, as dictated by his optimality condition, (24). As a result, the entrepreneur restricts the creation of new technology in the next period and can issue fewer patents, causing the price of new patents, p_t, to rise.

7 Closing the model

We now close the model by describing the behavior of households and aggregation of the model to account for the distribution of agents between households and entrepreneurs.

7.1 The household

The value of a patent derives from the monopoly profits accruing to the holder of the patent. We see this represented in the evolution of the assets held by a representative household. At the beginning of a period, the household is endowed with labor, l_h, for which it receives a competitive wage, w_t. At the beginning of the period, the household also holds a portfolio of assets, held between capital, $(k_t - n_t)$,[15] and patents, valued at the end of the previous period, $p_{t-1}t_t$. These investment goods can be used for three purposes: investment in the production of consumption goods ($k_{Y,t}$), holding current patents (t_t), and investment in the production of new technology ($k_{a,t} - n_t$). Once the returns from each of these projects and the returns to labor have been realized, the household determines how much of his return he wishes to consume (c_t) leaving the rest in the form of patents or capital. The evolution of the household's assets therefore evolve according to the equation:

$$\begin{aligned}(k_{t+1} - n_{t+1})\left(\frac{\eta}{1-\eta}\right) + p_t t_{t+1} = \\ &= w_t l_h + (1-\delta+r_t)k_{Y,t} + q_t \zeta k_{a,t}^{1-\lambda} A_t^{\lambda} g(\bar{\theta}_t)\left(\frac{\eta}{1-\eta}\right) + (p_t + \pi_t)t_t - c_t \\ &= w_t l_h + (1-\delta+r_t)\left(k_{Y,t} + (k_{A,t} - n_t)\left(\frac{\eta}{1-\eta}\right)\right) + (p_t+\pi_t)t_t - c_t \qquad (30)\end{aligned}$$

[15] k_t represents the overall level of capital, so the amount held by the household is $k_t - n_t$.

for the case of the debt contract. The second line takes into account the participation constraint, (10), which guarantees the household's share of the technology project to receive the market interest rate.

For the equity contract, the household's assets evolve similarly:

$$
\begin{aligned}
&(k_{t+1} - n_{t+1}) \left(\frac{\eta}{1-\eta}\right) + p_t t_{t+1} = \\
&\qquad = w_t l_h + (1-\delta+r_t) k_{Y,t} + s_t q_t \zeta k_{a,t}^{1-\lambda} A_t^{\lambda} \left(\frac{\eta}{1-\eta}\right) + (p_t + \pi_t) t_t - c_t \\
&\qquad = w_t l_h + (1-\delta+r_t) \left(k_{Y,t} + (K_{a,t} - n_t)\left(\frac{\eta}{1-\eta}\right)\right) + (p_t + \pi_t) t_t - c_t \qquad (31)
\end{aligned}
$$

The household makes its choices via a standard optimization problem:

$$
\max_{c_t} \mathbf{E}_t \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\sigma} - 1}{1-\sigma}\right)
$$

$$
\text{s.t. } k_{Y,t+1} + p_t t_{t+1} + i_{h,t+1} = w_t l_h + (1-\delta+r_t)(k_{Y,t} + i_{h,t}) + (p_t + \pi_t) t_t - c_t \qquad (32)
$$

where we define $i_{h,t} \equiv (k_{a,t} - n_t)\left(\frac{\eta}{1-\eta}\right)$ as the household's loan to the entrepreneur. The optimization gives the following Euler equations:

$$
c_t^{-\sigma} = \beta \mathrm{E}_t c_{t+1}^{-\sigma} (1 - \delta + r_{t+1}); \qquad (33)
$$

$$
c_t^{-\sigma} = \beta \mathrm{E}_t c_{t+1}^{-\sigma} \frac{p_{t+1} + \pi_{t+1}}{p_t}; \qquad (34)
$$

where the first Euler equation is the result for optimizing with respect to $k_{Y,t}$. The quantity of i_h, t held by the household is determined not by optimization, but by the availability of this asset as determined by the entrepreneur's optimization, (21).

7.2 Aggregation

As we are concerned with the progression of the economy given the existence of venture capitalists, we stipulate a constant proportion of the population, η, as entrepreneurs. We are not concerned with the progression of how this sector transforms over time, but with the effect of the sector on the rest of the economy.

Patents are held only by households. Entrepreneurs' assets are technically patents and priced as such, but they are sold immediately in order to fund a new round of technology investment. As a result, the total number of patents accumulating profit streams is simply the aggregate of household patents:

$$T_t = (1-\eta)t_t \tag{35}$$

However, each period, new patents are created by the entrepreneurs' projects. The patents at the end of the period for the debt contract are given by,

$$\begin{aligned} T_{t+1} &= \eta\left[q_t\zeta k_{a,t}^{1-\lambda}A_t^{\lambda}f(\bar{\theta}_t)\right] + (1-\eta)\left[t_t + q_t\zeta k_{a,t}^{1-\lambda}A_t^{\lambda}g(\bar{\theta}_t)\left(\frac{\eta}{1-\eta}\right)\right] \\ &= T_t + q_t\zeta K_{A,t}^{1-\lambda}A_t(1-\mu\Phi(\bar{\theta}_t)) \end{aligned} \tag{36}$$

which is similar to (6), except that it incorporates the rate patents are created from new technology and the destruction of value through the costly state verification. We also have a similar equation for the equity contract:

$$\begin{aligned} T_{t+1} &= \eta\left[q_t\zeta k_{a,t}^{1-\lambda}A_t^{\lambda}(1-s) - as_t^2\right] + (1-\eta)\left[t_t + q_t\zeta k_{a,t}^{1-\lambda}A_t^{\lambda}s_t\left(\frac{\eta}{1-\eta}\right)\right] \\ &= T_t + q_t\zeta K_{A,t}^{1-\lambda}A_t(1-as_t^2) \end{aligned} \tag{37}$$

We can write the overall level of labor as, $L_t = (1-\eta)l_t + \eta x_t$. Similarly, household consumption is given by $C_t = (1-\eta)c_t$. Since the entrepreneur holds his assets only in patents, household capital is divided between $K_{Y,t} = (1-\eta)k_{Y,t}$ and $K_{A,t} = (1-\eta)k_{A,t}$, with $K \equiv K_{Y,t} + K_{A,t} = 1$[16]

Finally, we are able to define a competitive equilibrium for both the debt and equity cases:

Definition 1 (Debt-based Decentralized Equilibrium)**.** *A Debt-based Decentralized Equilibrium consists of:*

1. *Households choosing a consumption and asset bundle, c_t, k_t, and t_t, to optimize their discounted lifetime utility with respect to their budget, as in (30),*

2. *Entrepreneurs choosing a consumption and asset bundle, c_t^e, and z_t, to optimize their discounted lifetime utility subject to their budget, (15),*

3. *Intermediate goods producers choosing an optimum amount of their intermediate good, x_t, to produce yielding a profit given by (3),*

4. *Final goods producers choosing the optimum level of output, y_t, paying competitive prices, r_t, w_t, determined by (2), and the marginal returns to production for L_t, respectively,*

5. *The technology, expressed as the number of intermediate capital goods used in production, evolving according to (6) with a new patent created for each new intermediate capital good,*

6. *Household labor and entrepreneurial endowments supplied inelastically at the beginning of each period, and*

7. *The overall allocation of $C_t, C_t^e, K_t, Y_t, T_t$, and Z_t, is feasible in that each element is non-negative and the allocation satisfies the market clearing conditions:*

$$K_{Y,t+1} + K_{A,t+1} = (1-\delta)K_{Y,t} + Y_t - C_t - C_t^e$$

   and (36), where C_t, C_t^e, and K_t are the sum of their respective individual variables of the same symbol.

[16]Recall that an individual entrepreneur's project defined in (5) used k_i units of capital. We assume capital is distributed evenly across the entrepreneurs' projects. Thus $k_{i,t} = K_{A,t}/\eta = \frac{(1-\eta)}{\eta}(k_{A,t})$.

Definition 2 (Equity-based Decentralized Equilibrium). *A Equity-based Decentralized Equilibrium consists of:*

1. *Households choosing a consumption and asset bundle, c_t, k_t, and t_t, to optimize their discounted lifetime utility with respect to their budget, as in (31),*

2. *Entrepreneurs choosing a consumption and asset bundle, c_t^e, and z_t, to optimize their discounted lifetime utility subject to their budget, (28),*

3. *Intermediate goods producers choosing an optimum amount of their intermediate good, x_t, to produce yielding a profit given by (3),*

4. *Final goods producers choosing the optimum level of output, y_t, paying competitive prices, r_t, w_t, determined by (2), and the marginal returns to production for L_t, respectively,*

5. *The technology, expressed as the number of intermediate capital goods used in production, evolving according to (6) with a new patent created for each new intermediate capital good,*

6. *Household labor and entrepreneurial endowments supplied inelastically at the beginning of each period, and*

7. *The overall allocation of $C_t, C_t^e, K_t, Y_t, T_t$, and Z_t, is feasible in that each element is non-negative and the allocation satisfies the market clearing conditions:*

$$K_{Y,t+1} + K_{A,t+1} = (1 - \delta)K_{Y,t} + Y_t - C_t - C_t^e$$

and (37), where C_t, C_t^e, and K_t are the sum of their respective individual variables of the same symbol.

The system of equations that make up our equilibrium conditions in the two decentralized equilibria are outlined in Appendix A.

8 Calibration

The model has been calibrated to match characteristics of the US economy. We match the long-term growth rate of 2% and use an interest rate of 8%. Capital depreciates at an annual rate of 15%, and our discount factor is 0.95 to represent annual data.

More important is our calibration for aspects specific to growth and finance. Our multiplier on the production of new technologies is 0.04, to allow a non-negligible amount of investment in these new technologies.This calibration allows for 15% of capital to be dedicated to the technology sector. Likewise, λ, our Cobb-Douglas parameter in the technology sector is 0.3, to provide a relatively low amount of curvature for this production function. This allows for large deviations in the amount of capital allocated to technology, increasing the effect of the mechanism we wish to model. For both equity and debt, we fix the steady state proportion of net-worth to overall investment in technology to 10%. This is roughly in line with US data, though the data tends to show a higher leverage for equity than for debt. We set them equal to isolate the effects of our mechanism from the effects of increased leverage.

Finally, all the results presented below are in terms of stationary variables. For variables that are non-stationary in the model, we have de-trended them using the now-endogenous growth rate.

9 Results

We study the effects of an unexpected positive technology shock. In a debt contract, the effect of such a shock will lead to higher returns for all projects. As the terms of the contract have already determined the level of $\bar{\theta}$, and the incentive compatibility constraint continues to bind, the returns going to the household from the successful technology projects will continue to be determined by the market interest rate. Thus, the majority of the windfall from a positive

Debt		Equity	
Variable	Value of Second Moment	Variable	Value of Second Moment
e^Z	0.1061	e^Z	0.1061
$\tilde{K}_a$	0.0146	$\tilde{K}_a$	0.0775
A	0.0005	A	0.0029
$\tilde{n}$	0.0042	$\tilde{n}$	0.0028
$g(\bar{\theta})$	0.0048	s	0.0398
$\tilde{c}^e$	0.0780	$\tilde{c}^e$	0.0505
r	0.0083	r	0.0084
pa	0.0671	pa	0.0696
$\tilde{Y}$	0.1416	$\tilde{Y}$	0.1412

Table 2: Variances of key variables in debt and equity contracts

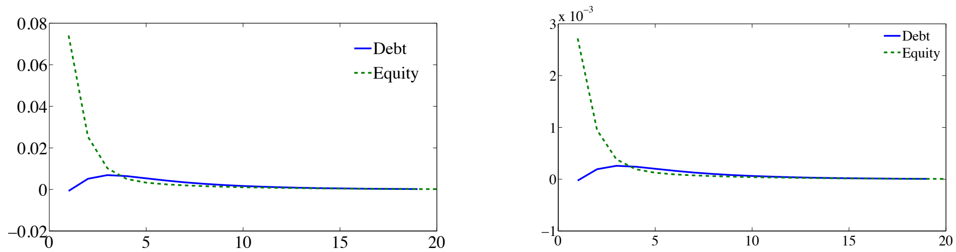
technology shock will accrue to the entrepreneur.[17] In contrast, the sharing of idiosyncratic risk in the equity contract also allows sharing of aggregate risk of technology fluctuations. The risk profile associated with the ability to share the windfall from technology fluctuations is the key difference between the two contracts.

Before discussing the transition following our shock, we first examine the magnitudes of the displacements caused by such technology shocks. To do this, we consider the variance of key variables given a simulation of 2,000 iterations of a technology shock for the debt contract and the equity contract, respectively. These magnitudes are given in Table 2. One can see that the magnitude of the shock, which we call e^Z, is equal for both simulations. This is by design, as we compare the responses to similar variations in technology growth. With this equivalent shock, the equity contract leads to a much higher deviation in the amount of capital dedicated to technology, which we can also see in Figure 2. In turn, this leads to a increase in the overall technology level roughly six times higher that of the debt contract. Note that we are referring to produced technology here, as the actual technology change will be a combination of the stochastic process, e^Z, and the endogenously produced technology, A. In our calibration, the stochastic element is stronger, while the relative comparison we make here is for the produced element alone. Thus, while we control for similar variances in the stochastic portion of total factor productivity, the

[17]There will be a slight increase to the household due to an unexpectedly low proportion of projects going bankrupt. However, given our calibration, the vast majority of the unexpected returns accrue to the entrepreneur. This also appears to be the case for most reasonable calibrations for the steady-state values of $\bar{\theta}_t$.

different responses of the technology sector to these deviations in each contract imply that the overall deviations in total factor productivity are not equal between contracts.

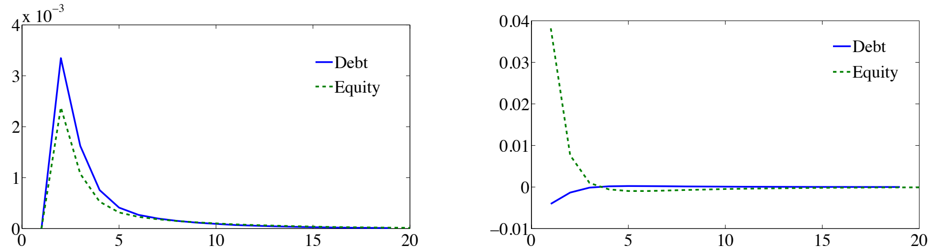
The entrepreneur is better off in the equity contract, too. While the entrepreneur receives more of the windfall in the debt contract, the increase in the amount raised from the household in the equity contract more than compensates for the increase in net-worth due to the windfall accruing to the entrepreneur under the debt contract.



(a) Impulse response for $K_{A,t}$ (b) Impulse response for A_t

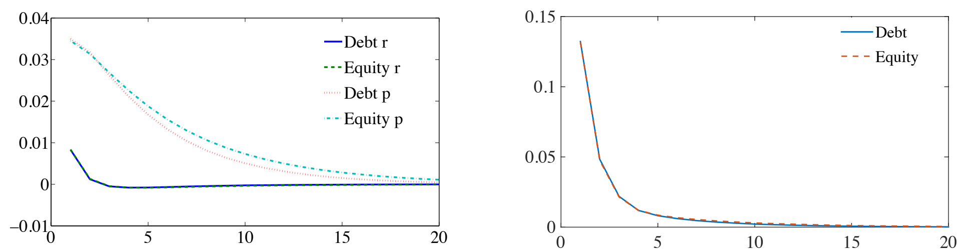
Figure 2: Response of contractual allocations to a technology shock

The left side of Figure 3 shows that the overall increase in net-worth for the investor is not much greater for the debt case, and the two contracts behave relatively similarly in the propagation of net-worth through time. In contrast, the share of the project accruing to the investor/lender varies greatly between the two contracts, as shown by the right side of the same figure. In the debt contract, an increase in the overall technology level and a subsequent increase in the entrepreneur's net-worth causes the entrepreneur to offer worse terms of investment for the lender. In contrast, the same fluctuations in the equity contract cause the entrepreneur to improve the terms of investment for the investor. This is because allowing for an improvement in the terms of investment for the household in the debt contract significantly changes the risk profile of the entrepreneur. Better terms for the household mean that the entrepreneur has to produce a higher amount before he even starts to make an income. After a positive technology shock, this effect is exaggerated by the increase in the market interest rate, due to an increased capital share in production, as shown in Figure 4. In contrast the equity contract allows the



(a) Impulse response for $\tilde{n}_t$ (b) Response of investor's/lender's share

Figure 3: Progression of the entrepreneur's behavior after a technology shock



(a) Returns after a technology shock (b) Output after a technology shock

Figure 4: Entrepreneur's behavior after a technology shock

entrepreneur to share both the risks of technology shocks and the rewards. As a result, the entrepreneur will be more inclined to allow for a higher share to accrue to the household after a technology shock, especially given that this improved terms of investment will allow for a much larger project overall. It is this risk-aversion in the debt contract that limits the increase in K_A to 19% of the increase in an equity contract following a positive technology development.

Finally, Figure 4 seems to imply that the production side of the economy is largely unaffected by the form of the contract in technology investment. While the price of patents remains slightly higher over time in an equity contract, the difference is very small relative to the overall deviation from the shock in the first place. More importantly, the interest rate and output rate are virtually identical between the two contracts.

However, the model simulated above has been de-trended to show how the economy returns to a steady-state after a stochastic shock. For non-stationary variables, we consider a ratio of variables with the same growth rate. Namely, we consider the ratio of the variable under question and the growth rate of technology, which drives the growth of the overall economy. Thus, Figure 4 actually implies that the model exhibits a co-movement of the ratio of output to technology. This is significant in that it means there is no extra growth in output under either contract due to the allocation of capital between sectors, but the gross growth in output itself is not equal under the two contracts. The equal co-movement in Figure 4 coupled with the higher increase in technology growth under the equity contract in Figure 2 imply that the output growth is actually five times greater in the equity contract. While a result of this magnitude is generally considered extreme in business cycle models, the economic growth associated with the development of personal computing and information technology in the 1980s and 1990s could correspond to such a large shift.

Furthermore, while we see that the returns to investment in the production sector is largely unchanged by the type of the contract in the technology sector, the response of the technology sector to such a shock changes direction when moving from an equity contract to a debt contract. As a result the correlation between the two reverses, in line with the fourth stylized fact in the introduction. This does come with one caveat: the variable which we use to represent technology development in this case only represents one composite element of total factor productivity; it does not represent the stochastic element. This caveat also applies to our explanation of total factor productivity variance. Thus, while we can show an underlying mechanism that would explain correspondence to such stylized facts, the magnitude of the mechanism relies on the calibration of the model to adjust the relative variance between the composite elements of total factor productivity. The key parameter in this regard is ζ, which we have set at 0.04, to dampen the effects of changes in the technology sector. The main justification of this was to allow a conservative result on the magnitude of the change from debt to equity, as 5x is already a startling result. In addition, the calibration of this model is very sensitive to changes in the value of ζ, so adjusting this parameter to match any particular stylized fact is inadvisable.

10 Conclusion

In a model in which technological investment occurs in a deliberate and incentivized way, we have shown that the type of financial contract plays a large role in both the dynamics of the development of new technology and in the magnitude of the response to independent technology shocks. A debt contract, characterized by borrowers' superior information of their own projects and their absorption of the macroeconomic risk, is characterized by relative stability at the cost of severely diminished quantities of capital raised. The equity contract, characterized by risk sharing with a preference for a small investor pool for efficient governance, allows total factor productivity to vary widely and closely follow the shocks to the economy.

These results are indeed in line with the stylized facts outlined in the introduction. First, the relative stability of total factor productivity during the 1960s corresponds to the use of the debt contract during this period, whereas the relatively high volatility in the 1980s corresponds with a move to equity.[18] Ultimately, our explanation for this is the absorption of risk by a relatively risk-loving and impatient agent through the debt contract in the 1960s.

Our model also corresponds with our second stylized fact, that the recession of 1973-74 was one characterized by relatively moderate yet persistent declines in utilization-adjusted total factor productivity. Again, the moderation is due to the absorption of risk by the entrepreneurs who borrow to fund their projects in the debt contract, while the duration is due to the reduction of these entrepreneurs' ability to lend following their absorption of a negative shock. This has the benefit of being the standard financial accelerator associated with previous analyses including this type of debt contract based on net-worth multipliers.

Most significantly, our model explains the large increase in investment in technological development that occurred in the 1980s. While the results of our model, a 5-fold increase in the investment to new technology, seems startling, it corresponds with the magnitude during this

[18]We make no claim for the 1990s as we do not wish to make any claims about whether technology shocks were abnormally stable in the period of "great moderation." Obviously, the first decade of this millennium is characterized by two significant events in venture capital, which is beyond the scope of this paper.

period. The lesson from this increase is simple: by allowing investors to share in the potential benefits from macroeconomic risk, their expected return from the growth created by investment in new technology spur an increase in the investment. Given the old adage, “a rising tide lifts all boats,” we add the caveat, “only if you allow people to put their boats in the water.” To abuse the metaphor, our model shows an additional effect that more boats in the water causes the rising tide to displace an even greater volume, benefiting each boat even further.

Finally, our model partially explains the divergence in total factor productivity and returns to investment in equity that occurred in the early 1980s. In our model, this is actually due to the negative correlation between produced technology and exogenous technology in the debt contract. As the real interest rate to investment in production ratio correlates positively with the exogenous technology process, a switch from a debt contract to an equity contract will result in a switch from a negative correlation between produced technology and return to production capital to a positive one. However, this explanation is only partial in that it only explains a switch in the correlation between return to production capital and *one* of the components of total factor productivity. The overall correlation would depend on the relative makeup of total factor productivity, which depends on the calibration of the model. This would require a more extreme calibration than the one we were prepared to offer.

Thus, we finally arrive at the question originally posed in this paper: were policy makers right in their decision to incentivize the move from debt to equity in technology investment? In the sense that the policy resulted in a dramatic increase in investment to technology, which had a positive effect on the development of digital technology, the answer is a resounding “yes.” However, our model shows that this increased investment comes at the price of increased volatility in total factor productivity and relative investment in technology. While the 1990s and 2000s were marked by economic conditions that make it difficult to assess whether this increased volatility was a factor, we maintain that our model can explain some variance in TFP of the 1980s.

Appendix A: Equilibrium Conditions

Below are the conditions for the closed form model with debt and equity, respectively. These are the equations used in the simulation. These are the conditions *before* de-trending, and capital letters indicate aggregate variables.

Debt:

The model starts with the aggregate production of technology, patents, and output:

$$A_{t+1} = \zeta(K_{A,t})^{1-\lambda}A_t^{\lambda} + A_t$$

$$T_{t+1} = q_t\zeta K_{A,t}^{1-\lambda}A_t^{\lambda}(f(\bar{\theta}) + g(\bar{\theta})) + T_t;$$

$$Y_t = e^{\Xi}L^{1-\alpha}K_{Y,t}^{\alpha}$$

where the difference between patents and technology is the change in value from the debt contract and the deadweight loss due to monitoring. We next have our market returns to capital and patents:

$$R_t = \alpha^2\frac{Y_t}{K_{Y,t}}$$

$$\Pi_t = (1-\alpha)\frac{Y_t}{A_t}$$

From here, we describe the contract through the participation constraint and the optimal conditions:

$$q_t\zeta K_{A,t}^{1-\lambda}A_t^{\lambda}g(\bar{\theta}) = (K_{A,t} - N_t)(1 + R_t - \delta)$$

$$1 + R_t - \delta = (1-\lambda)q_t\zeta K_{A,t}^{-\lambda}A_t^{\lambda}\left(1 - \mu\Phi(\bar{\theta}_t) + \mu\phi(\bar{\theta}_t)\frac{f(\bar{\theta}_t)}{f'(\bar{\theta}_t)}\right)$$

Next, our entrepreneur's budget and Euler equation are:

$$p_t Z_{t+1} = (x + Z_t p_t)^{1-\lambda} q_t \zeta \left(\frac{K_{a,t}}{N_t}\right)^{1-\lambda} A_t^{\lambda} f(\bar{\theta}_t) - C_t^e$$
$$p_t = \beta\gamma \mathbf{E}_t \left[p_{t+1} q_{t+1} \zeta \left(\frac{K_{a,t+1}}{N_{t+1}}\right)^{1-\lambda} A_{t+1}^{\lambda} f(\bar{\theta}_{t+1}) \right]$$

And the household's Euler equation is:

$$1 = \beta \mathbf{E}_t \left(\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} (1 + R_{t+1} - \delta) \right)$$
$$p_t = \beta \mathbf{E}_t \left(\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} (p_{t+1} + \Pi_{t+1}) \right)$$

Finally, we close the model with an aggregate budget constraint (the household's budget would also fit this purpose) and the evolution of our stochastic element.

$$K_{Y,t+1} + K_{A,t+1} = (1-\delta)(K_{Y,t}) + Y_t - C_t - C_t^e$$

$$\log(\Xi_t) = \sigma_{\Xi} \log(\Xi_{t-1}) + e_t;$$

Equity:

The model with the equity contract is essentially the same as the model with the debt contract. The only differences are the equations describing the contract and the behavior of the entrepreneur. Production and equilibrium returns are unchanged:

$$
\begin{aligned}
A_{t+1} &= \zeta(K_{A,t})^{1-\lambda}A_t^{\lambda} + A_t \\
T_{t+1} &= q_t\zeta K_{A,t}^{1-\lambda}A_t^{\lambda} - b\frac{K_{A,t}^2}{2} + T_t \\
Y_t &= e^{\Xi}L^{1-\alpha}K_{Y,t}^{\alpha} \\
R_t &= \alpha^2\frac{Y_t}{K_{Y,t}} \\
\Pi_t &= (1-\alpha)\frac{Y_t}{A_t}
\end{aligned}
$$

where the last two equations are determined by arbitrage. While the participation constraint differs from the debt contract, the only change is that the returns to the investor are no longer state dependent. Similarly, the optimality condition only differs in the second term on the right, which deals with the marginal cost of issuing more equity (the marginal benefits are the same in both cases, as are the opportunity costs for the funds).

$$
\begin{aligned}
s_t q_t\zeta K_{A,t}^{1-\lambda}A_t^{\lambda} &= (K_{A,t} - N_t)(1 + R_t - \delta) \\
1 + R_t - \delta &= (1-\lambda)q_t\zeta K_{A,t}^{-\lambda}A_t^{\lambda} - b(K_{A,t} - N_t)
\end{aligned}
$$

Likewise, since the entrepreneur views $\bar{\theta}$ as exogenous when making inter-temporal decisions under the debt contract, the entrepreneur's Euler equations do not include any marginal changes in $\bar{\theta}$. As such, the entrepreneur's inter-temporal optimality condition is the same as under debt, only replacing $f(\bar{\theta})$ with s_t.

$$
\begin{aligned}
p_t Z_{t+1} &= (X + p_t Z_t)^{1-\lambda}(1-s_t)q_t\zeta\left(\frac{K_{A,t}}{N_t}\right)^{1-\lambda}A_{t+1}^{-\lambda} - C_t^e \\
p_t &= \beta\gamma\mathbf{E}_t\left[p_{t+1}q_{t+1}(1-s_{t+1})\zeta\left[\frac{K_{A,t+1}}{N_{t+1}}\right]^{1-\lambda}A_{t+1}^{-\lambda}\right]
\end{aligned}
$$

Finally, the household's Euler equations, the aggregate budget, and the evolution of our stochastic variable are unchanged from the model under a debt contract.

$$1 = \beta \mathbf{E}_t \left(\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} (1 + R_{t+1} - \delta) \right)$$
$$p_t = \beta \mathbf{E}_t \left(\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} (p_{t+1} + \Pi_{t+1}) \right)$$
$$K_{Y,t+1} + K_{A,t+1} = (1 - \delta)(K_{Y,t}) + Y_t - C_t - C_t^e$$

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